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Dynamical Breaking of Supersymmetry and Its Restoration at High Temperatures

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The dynamical breaking of the supersymmetric Higgs model is discussed without adding the Fayet–Iliopoulos term to the Lagrangian. It is shown, in terms of the Nambu–Jona-Lasinio mechanism, that the supersymmetry breaking can be realized dynamically in the supersymmetric Higgs model. The supersymmetry behavior at finite temperatures is also investigated and it is shown that the supersymmetry broken dynamically at zero temperature can be restored at finite temperatures.

KEY WORDS: supersymmetry; dynamical breaking; NJL mechanism; finite temperature.

1. INTRODUCTION

The spontaneous breaking of supersymmetry and its behavior at finite temperatures has been investigated by many authors (Das and Kaku, 1978; Dicus and Tata, 1984; Fayet and Iliopoulos, 1974; Fuchs, 1984; Girardello *et al.*, 1981; Hove, 1982; Iliopoulos and Zumino, 1974; O'raifeartaigh, 1975; Salam and Strasdee, 1974). The usual method is to add a parity-violating term (Fayet and Iliopoulos, 1974) (Fayet–Iliopoulos term) to the Lagrangian. The adding of the Fayet–Iliopoulos term to the Lagrangian leads to mass splitting between bosons and fermions in the supermultiplets and, hence, the supersymmetry breaks down spontaneously.

A parallel development is the study of supersymmetry behavior at finite temperatures. Some authors have shown (Das and Kaku, 1978; Girardello *et al.*, 1981) that at zero temperature the supersymmetry is not easy to break spontaneously, but that finite temperatures automatically break the supersymmetry. They argued that the supersymmetry broken at zero temperature cannot be restored at finite temperatures. Finite temperatures always break supersymmetry.

However, since supersymmetry is so special, one would like to somehow maintain it at high temperatures. This prompted Hove (1982) to propose a modified

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definition of order parameters at finite temperatures and to examine supersymmetry behavior at finite temperatures. Unfortunately, this modified definition of order parameters still does not lead to the expected behavior of supersymmetry at finite temperatures.

Some time ago, however, some authors (Cahill, 1991; Kumar, 1990; Song and Xu, 1993a,b; Wang *et al.*, 2002; Xu and Song, 1992) have again studied supersymmetry braking and its behavior at finite temperatures. In Xu and Song (1992), Song and Xu (1993a,b), and wang *et al.* (2002), it is shown that the supersymmetry can be broken dynamically without adding the Fayet–Iliopoulos term to the Lagrangian, and that the supersymmetry broken dynamically at zero temperature can be restored at finite temperatures.

The purpose of the present paper is to investigate the behavior of the supersymmetric Higgs model (Fayet, 1976; Fayet and Ferrara, 1977) at finite temperatures in terms of the Nambu–Jona-Lasinio (NJL) mechanism (Nambu and Jona-Lasinio, 1961). The key point of our method is to establish the self-consistency equation for the order parameter at finite temperatures and to solve the self-consistency equation by putting in the momentum cutoff. We will show that the supersymmetry broken at zero temperature can be restored at a finite temperature.

The paper is organized as follows. In Section 2 we will briefly review the NJL theory and show, in terms of the NJL mechanism, that the contribution of the self-energy of the Dirac spinor and the gauge field in the supersymmetric Higgs model (with the parity-violating parameter $\xi = 0$) generate a nonvanishing vacuum expectation value of the order parameter and provide different masses to the bosons and the Dirac spinor in the supermultiplet and, hence, the supersymmetry breaks down dynamically.

In Section 3 we will investigate the behavior of the supersymmetry at finite temperatures and shown, by solving the self-consistency equation for the order parameter at finite temperatures, that the supersymmetry that is broken dynamically at zero temperature can be restored at a critical temperature $T_c = \sqrt{3}\Lambda/\sqrt{10}\pi$, where Λ denotes the momentum cutoff. Finally, we summarize our conclusions.

2. NJL MECHANISM AND DYNAMICAL BREAKING OF THE SUPERSYMMETRIC HIGGS MODEL

In 1961, Nambu and Jona-Lasinio suggested that the nucleon mass arises largely as a self-energy of some primary massless fermion fields, and regarded real nucleons as quasi-particle excitations. The Lagrangian density of the NJL model is given as

$$L = L_0 + L_I, \tag{1}$$

where the free Lagrangian L_0 is given by

$$L_0 = -\bar{\psi}\gamma_\mu\partial_\mu\psi \tag{2}$$

and L_I is a four-fermions interaction of the type

$$L_I = g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \tag{3}$$

Nambu and Jona-Lasinio introduced a self-energy term $\delta m\bar{\psi}\psi$ to the Lagrangian (1) and rewrite it as

$$L = (L_0 - \delta m\bar{\psi}\psi) + (L_I + \delta m\bar{\psi}\psi) = L_0' + L_I', \tag{4}$$

where

$$L_0' = -\bar{\psi}(\gamma_\mu\delta_\mu + \delta m)\psi, \tag{5}$$

$$L_I' = g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] + \delta m\bar{\psi}\psi. \tag{6}$$

From Eq. (5) we see that the crucial assumption of the NJL mechanism is that despite the vanishing of the bare fermion mass, the physical mass m of the fermion is nonzero. Since the bare fermion mass $m_0 = 0$, we have $m = \delta m$ and so we obtain the self-consistency equation for the physical mass of fermion:

$$m = \delta m = \sum(p), \tag{7}$$

in which $\sum(p)$ is the unrenoralized proper self-energy part of the fermion. Nambu and Jona-Lasinio have evaluated m from the self-energy diagram of the fermion to the first order in g. The result is

$$m \approx 2igTrS_F(0) = \int \frac{d^4p}{(2\pi)^4}\frac{8igm}{p^2+m^2} = \frac{gm}{2\pi^2}\left[\Lambda^2 - m^2 \ln\frac{\Lambda^2+m^2}{m^2}\right], \tag{8}$$

in which Λ^2 is the invariant momentum cutoff $p^2 = \Lambda^2$.

From above discussion we see that in the NJL theory, starting from zero-mass fermion, one generates the dynamical mass of fermion self-consistently.

Lurie and Macfarlane (1964) have shown the equivalence between Lagrangian field theory of four-fermion type considered by Nambu and Jona-Lasinio and a Lagrangian theory of the same fermion fields with coupling of Yukawa type

$$L_Y = L_0 + G\bar{\psi}\psi\Phi_S + G\bar{\psi}\gamma_5\psi\Phi_P \tag{9}$$

and obtained the physical fermion mass in the equivalent Yukawa theory in the same way.

In this paper we utilize the NJL mechanism mentioned above to discuss the dynamical breaking of the supersymmetric Higgs model (in which we put the parity-violating parameter $\xi = 0$). The model is based on the field theory describing the interaction of a vector multiplet $V = (V^\mu, \lambda, D)$ with a left-handed chiral multiplet $S = (A, B, \psi, F, G)$, where all fields are taken to be massless. All unexplained notation in the present paper can be found in Fayet (1976) and

Fayet and Ferrara (1977). After elimination of the auxiliary fields, the Lagrangian density (with $\xi = 0$) has the following form

$$L = -\frac{1}{4}V^{\mu\nu}V_{\mu\nu} - \frac{1}{2}i\bar{\lambda}\gamma^{\mu}\partial_{\mu}\lambda - i\bar{\psi}_L\gamma^{\mu}D_{\mu}\psi_L - D^{\mu}\phi^{\dagger}D_{\mu}\phi$$
$$+ ie\sqrt{2}(\bar{\psi}_L\lambda\phi + \phi^{\dagger}\bar{\lambda}\psi_L) - \frac{1}{2}|e\phi^{\dagger}\phi|^2, \tag{10}$$

where

$$V_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}, \quad D_{\mu} = \partial_{\mu} + ieV_{\mu}, \quad \phi = -i(A - iB)/\sqrt{2}. \tag{11}$$

Without loss of generality we can choose the electric charge $e > 0$.

We define a Dirac spinor $E = \psi_L + \lambda_R$, and choose a gauge where $A = 0$. Then the Lagrangian (10) reads

$$L = \frac{1}{4}V^{\mu\nu}V_{\mu\nu} - i\bar{E}\gamma^{\mu}\partial_{\mu}E - \frac{1}{2}\partial^{\mu}B\partial_{\mu}B - \frac{1}{2}(eB)^2V^{\mu}V_{\mu}$$
$$- \frac{1}{8}\left(eB^2\right)^2 - i(eB)\bar{E}E + e\bar{E}_L\gamma^{\mu}V_{\mu}E_L. \tag{12}$$

It describes the self-interaction for a vector multiplet $V = (V^{\mu}, E, B)$.

In Eq. (12), if the vacuum expectation value of the real scalar field B is nonvanishing, both of the gauge symmetry and the supersymmetry will be broken down. We denote the vacuum expectation of the real scalar field as

$$\langle B\rangle = v. \tag{13}$$

Translating the scalar field B as

$$B \to B + v, \tag{14}$$

this translating changes the quadratic terms in the Lagrangian (12) into the following form

$$-\frac{1}{2}(ev)^2V^{\mu}V_{\mu} - \frac{3}{4}(ev)^2B^2 - i(eV)\bar{E}E, \tag{15}$$

and from which we obtain the masses of the gauge field V^{μ}, the Dirac spinor E, and the real scalar field B as

$$m_V = m_E = ev, \quad m_B = \sqrt{3/2}ev. \tag{16}$$

which means that either the gauge symmetry or the supersymmetry is broken.

We now introduce the external source J_B coupled to the real scalar field B, then the Lagrangian density becomes

$$L[J] = L + J_B B \tag{17}$$

and the equation of motion following from the Lagrangian (17) is given by

$$\Box B - e^2 V^\mu V_\mu B - \frac{1}{2} e^2 B^3 - ie\bar{E}E = 0. \tag{18}$$

As usual, the generating functional is given by

$$Z[J] = \int [d\varphi] \exp\left(\frac{i}{\hbar} \int d^4x L[J]\right), \tag{19}$$

where $[d\varphi]$ stands generically for all fields. By using the generating functional $Z[J]$, the vacuum expectation value of the real scalar field B, in the presence of the external source can be found as

$$\langle B \rangle^J = \frac{\delta W[J]}{\delta J_B} = \frac{1}{Z[J]} \int [d\varphi] B \, \exp\left(\frac{i}{\hbar} \int d^4x L[J]\right), \tag{20}$$

with

$$W[J] = (\hbar/i) \ln(Z[J]). \tag{21}$$

Let us now take the vacuum expectation value of Eq. (18). In the limit $J_B \to 0$, to the lowest-order approximation in $\hbar$, we obtain the self-consistency equation as

$$\frac{1}{2} e^2 v^3 + e^2 \langle V^\mu V_\mu \rangle v + ie\langle \bar{E}E \rangle = 0. \tag{22}$$

After the momentum cutoff $p^2 = \Lambda^2$ in the momentum integral, the expectation values $\langle V^\mu V_\mu \rangle$ and $\langle \bar{E}E \rangle$ in Eq. (22) will be finite quantities. We find

$$\begin{aligned}
\langle \bar{E}E \rangle &= -Tr S_F(0) = 4m_E \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + m_E^2} \\
&= \frac{im_E}{2\pi^2} \left[\Lambda \sqrt{\Lambda^2 + m_E^2} - m_E^2 \, \ln \frac{\Lambda + \sqrt{\Lambda^2 + m_E^2}}{m_E} \right], \\
\langle V^\mu V_\mu \rangle &= g^{\mu\nu} D_{\mu\nu}(0) = \int \frac{d^4p}{(2\pi)^4} \frac{-ig^{\mu\nu}}{p^2 + m_V^2} \left(g_{\mu\nu} + \frac{p_\mu p_\nu}{m_V^2} \right) \\
&= \frac{3}{8\pi^2} \left[\Lambda \sqrt{\Lambda^2 + m_v^2} - m_V^2 \, \ln \frac{\Lambda + \sqrt{\Lambda^2 + m_V^2}}{m_V} \right].
\end{aligned} \tag{23}$$

Substituting Eqs.(16) and (23) into the self-consistency equation (22), we have

$$v^2 = \frac{1}{4\pi^2} \left[\Lambda \sqrt{\Lambda^2 + e^2 v^2} - e^2 v^2 \, \ln \frac{\Lambda + \sqrt{\Lambda^2 + e^2 v^2}}{ev} \right]. \tag{24}$$

From this equation, we can finally obtain the nonvanishing vacuum expectation value of the real scalar field. Since v is determined by the self-energy of the Dirac spinor and the gauge field, we conclude that either the gauge symmetry or the supersymmetry breaks down dynamically.

3. THE RESTORATION OF GAUGE SYMMETRY AND SUPERSYMMETRY AT FINITE TEMPERATURES

In the previous section we have established the self-consistency equation for the order parameter $\langle B \rangle = v$ and examined dynamical breaking of gauge and supersymmetry. If a corresponding equation at finite temperatures can be established then we can investigate gauge and supersymmetry behavior at finite temperatures by solving the self-consistency equation. We wish to find such a temperature T_c at which the vacuum expectation v reduces to zero and the gauge symmetry and the supersymmetry will be restored again.

It is well known that, at finite temperatures, all physically interesting quantities such as Green's functions in a system are given not by the vacuum-to-vacuum transition amplitude as in the usual field theories, but by the statistical average defined by (Dolan and Jackiw, 1974)

$$G_\beta(x_1, x_2, \ldots, x_n) = \frac{Tr\{\exp(-\beta H) T[\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)]\}}{Tr\ \exp(-\beta H)} \tag{25}$$

where β is proportional to the inverse of temperature and H denotes the Hamiltonian of the system. In the field theory the Green function at finite temperatures (Bernard, 1974) can be written as

$$G_\beta(x_1, x_2, \ldots, x_n) = \frac{1}{z^\beta[J]} \frac{\delta^n Z^\beta[J]}{\delta J(x_2)\cdots\delta J(x_n)}, \tag{26}$$

where $Z^\beta[J]$ is the generating functional at finite temperatures

$$Z^\beta[J] = \int [d\varphi] \exp\left\{\int_0^\beta d\tau \int d^3x L_E[J]\right\}. \tag{27}$$

Here $L_E[J]$ denotes the Lagrangian $L[J]$ in the Euclidean space. Here and afterwards we take $\hbar = 1$.

The important observation in field theory at finite temperatures is the fact that the finite-temperature Green's functons satisfy the same differential equations as the zero-temperature Green's functions except that they satisfy a periodic (antiperiodic for the fermion case) boundary condition for an imaginary time τ, and the momentum $p = (p_0, \vec{p})$ has to be replaced by

$$p = (\omega_n, \vec{p}), \tag{28}$$

with

$$\omega_n = (2n+1)\pi/\beta, \quad (n \text{ integer for the fermions}), \tag{29}$$

$$\omega_n = 2n\pi/\beta, \quad (n \text{ integer for the bosons}). \tag{30}$$

So, at finite temperatures, by using Eq. (23), the self-consistency equation (22) changes to

$$e^2 v^2(\beta) = I_1 + I_2, \tag{31}$$

with

$$I_1 = -\frac{6e^2}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{(2n\pi/\beta)^2 + \vec{p}^2 + e^2 v^2(\beta)} = I_1' + I_1'', \tag{32}$$

$$I_2 = \frac{8e^2}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{[(2n+1)\pi/\beta]^2 + \vec{p}^2 + e^2 v^2(\beta)} = I_2' + I_2'', \tag{33}$$

in which

$$I_1' = -\frac{3e^2}{2\pi^2} \int_0^\infty \frac{|\vec{p}|^2 d|\vec{p}|}{(|\vec{p}|^2 + e^2 v^2(\beta))^{1/2}}, \tag{34}$$

$$I_1'' = -\frac{3e^2}{\pi^2} \int_0^\infty \frac{|\vec{p}|^2 d|\vec{p}|}{(|\vec{p}|^2 + e^2 v^2(\beta))^{1/2} \{\exp[\beta(|\vec{p}|^2 + e^2 v^2(\beta))^{1/2}] - 1\}}, \tag{35}$$

$$I_2' = \frac{2e^2}{\pi^2} \int_0^\infty \frac{|\vec{p}|^2 d|\vec{p}|}{(|\vec{p}|^2 + e^2 v^2(\beta))^{1/2}}, \tag{36}$$

$$I_2'' = -\frac{4e^2}{\pi^2} \int_0^\infty \frac{|\vec{p}|^2 d|\vec{p}|}{(|\vec{p}|^2 + e^2 v^2(\beta))^{1/2} \{\exp[\beta(|\vec{p}|^2 + e^2 v^2(\beta))^{1/2}] + 1\}}. \tag{37}$$

The integrations in Eqs. (34) and (36) are divergent. As before, introducing a momentum cutoff Λ one can make the integration finite. The result is

$$I_1' = -\frac{3e^2}{4\pi^2} \left[\Lambda\sqrt{\Lambda^2 + e^2 v^2(\beta)} - e^2 v^2(\beta) \ln \frac{\Lambda + \sqrt{\Lambda^2 + e^2 v^2(\beta)}}{ev(\beta)} \right], \tag{38}$$

$$I_2' = \frac{e^2}{\pi^2} \left[\Lambda\sqrt{\Lambda^2 + e^2 v^2(\beta)} - e^2 v^2(\beta) \ln \frac{\Lambda + \sqrt{\Lambda^2 + e^2 v^2(\beta)}}{ev(\beta)} \right]. \tag{39}$$

We are interested in the supersymmetry behavior at high temperatures and wish to find the critical temperature T_c at which the order parameter $\langle B(\beta) \rangle = v(\beta)$

tends to zero. So the integrations in Eqs. (35) and (37) can be calculated in the approximation $v(\beta) = 0$. In this approximation the integrations in Eqs. (35) and (37) turn out to be

$$I_1'' = \frac{3e^2}{\pi^2}\int_0^\infty \frac{|\vec{p}|d|\vec{p}|}{1-\exp(\beta_c|\vec{p}|)} = -\frac{e^2}{2\beta_c^2}, \tag{40}$$

$$I_2'' = -\frac{4e^2}{\pi^2}\int_0^\infty \frac{|\vec{p}|d|\vec{p}|}{1+\exp(\beta_c|\vec{p}|)} = -\frac{e^2}{3\beta_c^2}. \tag{41}$$

Substituting Eqs. (38)–(41) into Eq. (31) and taking the limit $v(\beta_c) \to 0$, one gets

$$T_c = \frac{1}{\beta_c} = \sqrt{\frac{3}{10}\frac{\Lambda}{\pi}}. \tag{42}$$

Thus, we have found the critical temperature T_c at which the order parameter $\langle B(\beta)\rangle$ tends to zero, and $m_V = m_E = m_B = 0$, which means that the dynamical breaking of the gauge symmetry and the supersymmetry at zero temperature can be restored.

The gauge symmetry and the supersymmetry restoration at finite temperatures can also be shown from the statistical average of the auxiliary fields behavior at finite temperatures. Following from Fayet (1976) and Fayet and Ferrara (1977) (with $\xi = 0$), we can obtain the auxiliary fields as

$$F = G = 0, \quad D = -\frac{1}{2}eB^2. \tag{43}$$

So at finite temperatures, the vacuum expectation of the auxiliary fields changes to

$$\langle F(\beta)\rangle = \langle G(\beta)\rangle = 0, \quad \langle D(\beta)\rangle = -\frac{e}{2}v^2(\beta). \tag{44}$$

We see from Eq. (44) that, when $T \to T_c$, $v(\beta) \to 0$. So the statistical average of the auxiliary fields tend to zero at the critical temperature. This is another sign of the supersymmetry restoration at finite temperatures.

In the model of Das and Kaku (1978), the vacuum expectation value of the auxiliary field D is given by

$$\langle D\rangle = -\frac{e}{2}v^2 - \xi, \tag{45}$$

where v is the vacuum expectation of the scalar field B and ξ denotes the coefficient of the Fayet–Iliopoulos term. We see from Eq. (45) that at finite temperatures $\langle D(\beta)\rangle$ does not vanish when $v(\beta) = 0$, because ξ is temperature-independent. Thus, in the model of Das and Kaku (1978) the supersymmetry cannot be restored at finite temperatures.

4. CONCLUSIONS

The dynamical breaking of the gauge symmetry and the supersymmetry is studied in terms of the NJL mechanism. Starting from the supersymmetric Higgs model that involves a vector multiplet and a left-handed chiral multiplet in its Lagrangian, we have shown that the contribution of self-energy part of the Dirac spinor and the gauge vector to the self-consistency equation for the order parameter generate different dynamical masses for different fields and leads to mass splitting between bosons and fermion in the supermultiplet, and, hence, the gauge symmetry and the supersymmetry break down dynamically at zero temperature. We have shown, by solving the self-consistency equation for the order parameter at finite temperatures, that the gauge symmetry and the supersymmetry that is broken dynamically at zero temperature can be restored at a critical temperature.

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